6.2 FACTORING TRINOMIALS:

(1) FACTOR TRI'S WHEN A = 1

NOTE: STD FORM TRI: $AX^2 + BX + C$

A) USE FOIL FOR MULTIPLYING: $(X+3)(X-5)$ (F, L, I, O)

=

B) NOW HOW CAN WE GO "BACKWARDS" & FACTOR A TRINOMIAL?

$= X^2 + -5X + 3X - 15$

$= X^2 - 2X - 15$

(FIRST)(LAST)(OUTER)(INNER)

C) USE "REVERSE" FOIL OR FLOI to FACTOR

STEPS:

EXAMPLE

1ST, 2ND (MIDDLE), 3RD (LAST) ← TERM

1) IS THERE A GCF to PULL OUT? $X^2 - 2X - 15$ → $-3, 5$; $-1, 15$

4) L ↑

2) MAKE ( )( ) ( )( )

3) F: WHAT TIMES WHAT GIVES 1ST TERM? $(X \quad)(X \quad)$

4) L: LIST FACTORS OF LAST TERM

$(X - 1)(X + 15)$

$-1X$

$+ \; +15X$

$-14X$: DOES NOT $= -2X$ TRY AGAIN

5) OI: UNLOCK SAFE!

DOES THE OUTER + INNER = MIDDLE TERM

$(X - 3)(X + 5)$

$-3X$

$+ \; 5X$

$2X$ NO! UNLOCK SAFE

I: $3X$

O: $-5X$

$+ \; -2X$ = MIDDLE TERM ✓

$(X+3)(X-5)$

$3X$

6.2 CONT.

EX 1: (1st) (2nd) (3rd)

A) $Y^2+2Y-35$ 1) IS THERE GCF? (NO)

$(\quad)(\quad)$ 2)

$(Y\quad)(Y\quad)$ F: 3) Y · Y ARE FACTORS OF 1st TERM

$(Y-5)(Y+7)$ L: 4) $-35 \rightarrow -5 \cdot 7$ ✱ , $-1 \cdot 35$

HINT: PICK FACTORS CLOSE TO EACH OTHER

$-5Y$

$7Y$

$+$

$2Y$ IS THIS 2nd TERM?

OI: 5) O: 7Y
+ I: −5Y

+2Y: MATCHS MIDDLE TERM

YES: SAFE IS OPEN!

$(Y-5)(Y+7)$ IS CORRECT FACTORS!!

EX 2: $M^2+6M+7 \rightarrow 1 \cdot 7$, $-1 \cdot -7$ (MIDDLE TERM)

$(M\quad)(M\quad)$ F: 3)

L: 4) 3RD TERM $7 = 1 \cdot 7$, $= -1 \cdot -7$

$(M+1)(M+7)$

$1M$

$7M$

$+8M$ DOES NOT MATCH MIDDLE TERM

TRY AGAIN

$(M-1)(M-7)$

$-1M$

$-7M$

$-8M$ ✗ AGAIN, DOES NOT = MIDDLE TERM ∴ CANNOT BE FACTORED PRIME!!

6.2

EX 3: $(P^2 + 6AP - 16A^2)$ (↓ MIDDLE) → $2A \cdot -8A$ (L)
$+8A \cdot -2A$

F: $(P \quad)(P \quad)$
L: $(P+2A)(P-8A)$ — $2AP$
OI: $+ \; -8AP$
$-6AP$ ✗ NO GOOD! TRY AGAIN.

✓ $(P-2A)(P+8A)$ — $-2AP$
$+ \; 8AP$
$6AP$ ✓ UNLOCKED SAFE! = MIDDLE TERM.

EX 4: $16Y^3 - 32Y^2 - 48Y$ IS THERE GCF? YES!

pull out GCF! → $16Y(Y^2 - 2Y - 3)$ —— $-1, 3$: $-3 = -1 \cdot 3$
RULE #1

$16Y(Y-1)(Y+3)$ — $-1Y$
$+ \; 3Y$
$2Y$ ✓ MATCHES MIDDLE TERM!

(2) FACTORING WHEN $A \neq 1$

✓ DO EX 6 1st

A B C

EX 5: $12R^2 - 5R - 2$ F: HUNCH IS $3 \cdot 4$ WORKS (FACTORS ARE CLOSE TO EACH OTHER)
NOW VERIFY:
~~$AC = 12(-2) = -24$~~

F: $(3R \quad)(4R \quad)$
L: $(3R-1)(4R+2)$ L: $-2 \rightarrow -1 \cdot 2$, $1 \cdot -2$] ORDER MATTERS!
OI: $-4R$
$+ \; 6R$
$2R$ ✗ $\neq -5R$ TRY AGAIN!

$(3R+1)(4R-2)$
OI: $4R$, $-6R$, $-2R$ NO $\neq -5R$

✓ $(4R+1)(3R-2)$ — $3R$, $-8R$, $-5R$ ✓ $= -5R$

(L)

6.2 CONT.

EX 6A: $3X^2+7X+2$ → 1·2 (ORDER MATTERS)

F: $(3X+2)(X+1)$ — inner: 2X, outer: 3X; + 5X ≠ 7X

✱ $(3X+1)(X+2)$ — inner: 1X, outer: 6X; + 7X = 7X ✓

EX 7: $18M^2-19MX-12X^2$ (L) -3·4 | -6·2
4·-3 | 6·-2

(F) 2·9 or 6·3

1) NO GCF
2) MANY FACTORS OF 1ST TERM

LOOK AT EXAMPLE 7 IN BOOK!
THIS IS TRIAL & ERROR (PIA!)

TRY F: $(9M+4X)(2M-3X)$
L: 8XM
OI: -27XM
+
-19XM = MIDDLE TERM ✓

DON'T LIKE THAT

EX 8: $-3X^2+16X+12$ LOOK A < 1: MULTIPLY (-1)

$3X^2-16X-12$ (MULTIPLY) BY (-1) OR SWITCH SIGNS

2·-6

F $(3X+2)(X-6)$ — inner: 2X, outer: -18X; + -16X = -16X

NOTE THIS TECHNIQUE IS DIFFERENT FROM BOOK

SUBST. TECHNIQUE:

EX 10: $2(X+3)^2+5(X+3)-12$

LET $T=(X+3)$ $\quad 2T^2+5T-12$

$=(2T-3)(T+4)$

SUBST. BACK IN!

$\rightarrow [2(X+3)-3][(X+3)+4]$

$=[2X+6-3][X+7]$

$=[2X+3][X+7]$

6.3 SPECIAL FACTORING

① FACTOR A DIFFERENCE OF SQUARES: MUST KNOW!

$(1^{ST})^2 - (2^{ND})^2 = (1^{ST}+2^{ND})\cdot(1^{ST}-2^{ND})$

1) CAN YOU GET THE PERFECT SQ. RT OF EACH TERM?
2) DO YOU HAVE − (SUBST)!

EX 1:

A) t^2-36 $\sqrt{t^2}=t$; $\sqrt{36}=6$ YES!

$(t)^2-(6)^2=(t+6)(t-6)$

C) $4A^2-64$ $\sqrt{4A^2}=2A$ $\sqrt{64}=8$ YES!

$(2A)^2-(8)^2=(2A+8)(2A-8)$

F) X^4+4 $\sqrt{X^4}=X^2$ $\sqrt{4}=2$ YES BUT $+^{(1)}$

$(X^2)^2+(2)^2=$ PRIME CAN'T FACTOR!!

② FACTOR A PERFECT SQUARE TRINOMIAL:

$(1^{ST})^2+2(1^{ST})(2^{ND})+(2^{ND})^2=(1^{ST}+2^{ND})^2$

$(1^{ST})^2-2(1^{ST})(2^{ND})+(2^{ND})^2=(1^{ST}-2^{ND})^2$

IS $4m^2+20m+25$ A PERFECT SQ. TRI?

$=(2m)^2+(2m)(5)(2)+(5)^2=(2m+5)^2$ ✓

ASK: ① CAN YOU GET PERFECT SQ. RT FOR 1ST & 3RD TERM?

$\sqrt{4m^2}=2m$; $\sqrt{25}=5$ YES

② DOES MULTIPLY THE PERFECT SQ. RT'S & DOUBLE = ↑ MIDDLE TERM?

$(2m)(5)\cdot 2=20m$ YES! ∴ FACTOR

6.3 CONT. P2

EX 2: $144P^2 - 120P + 25$; $\sqrt{144P^2}$ $\sqrt{25}$ DOUBLE

A) $(12P \cdot 5) \cdot 2 = 120P$

$= (12P - 5)^2$

MATCHES MIDDLE TERM ∴ YES

B) $4m^2 + 20mn + 49n^2$; $\sqrt{4m^2}$ $\sqrt{49n^2}$

CAN NOT BE FACTORED $(2m \cdot 7n)2 = 28mn$

PRIME $\neq 20mn$ (MIDDLE TERM)

DIFF OF SQ'S

D) $(m^2 - 8m) + (16 - P^2)$: GROUPING!

GCF → $m \cdot (m - 8) \cdot (4 - P)(4 + P)$: DIFFERENT THAN BOOKS ANSWERS BUT SAME!

(3) Factor DIFFERENCE OF CUBES:

$$X^3 - Y^3 = (X - Y)(X^2 + XY + Y^2)$$

CHECK: $(m)^3 - (2)^3$

EX 3A: $m^3 - 8$ $\sqrt[3]{m^3} = m$ $\sqrt[3]{8} = 2$ HAVE PERFECT CUBIC ROOTS

$(m - 2) \cdot ((m)^2 + (2)(m) + (2)^2)$

$= (m - 2)(m^2 + 2m + 4)$

SIDE NOTE: $\sqrt[3]{X^9} = X^3$

EXP ÷ BY 3 $\sqrt[3]{X^{12}} = X^4$

WATCH SIGN! IT'S (−)

(4) Factor SUM OF CUBES:

$$X^3 + Y^3 = (X + Y)(X^2 - XY + Y^2)$$

EX 4A: $r^3 + 27$ $\sqrt[3]{r^3} = r$ $\sqrt[3]{27} = 3$

$(R)^3 + (3)^3 = (R + 3)(R^2 - 3R + 9)$ → $3^2 = 9$

D) $3X^3 + 192$ $\sqrt[3]{X^3} = X$ $\sqrt[3]{64} = 4$

GCF → $3(X^3 + 64) = 3(X + 4)(X^2 + 4X + 4^2)$

RULE #1!! $= 3(X + 4)(X^2 + 4X + 16)$